

Natural Split Mechanism for Sfermions: $N=2$ Supersymmetry in Phenomenology

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Abstract

We suggest a natural split mechanism for sfermions based on $N=2$ supersymmetry (SUSY). $N=2$ SUSY protects a sfermion in an $N=2$ multiplet from gaining weight by SUSY breaking. Therefore, if partly $N=2$ SUSY is effectively obtained, a split spectrum can be realized naturally. As an example of the natural split mechanism, we build a gauge-mediated SUSY breaking-like model assuming $N=2$ SUSY is partly broken in an underlying theory. The model explains the Higgs boson mass and muon anomalous magnetic dipole moment within 1σ level with a splitting sfermion spectrum. The model has seven light sparticles described by three free parameters and predicts a new chiral multiplet, sb: the $N=2$ partner of the $N=1$ $U(1)_Y$ vector multiplet. The bini, the fermion component of the sb, weighs MeVs. We mention the experimental and cosmological aspects of the model.

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1 Introduction

After the discovery of the Standard Model (SM) Higgs-like boson at the Large Hadron Collider (LHC) [1], no new particle has been found. On the other hand, some deviations from the SM have been observed in flavor physics [2]. If nature has supersymmetry (SUSY), the current experiments suggest an interesting possibility: a split spectrum for sfermions. No discovery of sparticles [3] gives bounds to the squark masses: the squarks should be heavier than $\mathcal{O}(1)$ TeV. This is consistent with the simplest SUSY SM (minimal supersymmetric standard model: MSSM) prediction, since stops are required to be heavier than $\mathcal{O}(10)$ TeV to explain the Higgs boson mass of about 125 GeV unless the SUSY breaking stop-Higgs trilinear coupling is parametrically large [4]. In the case, the heavy sfermions automatically solve/alleviate the flavor/CP problem, the proton decay problem, several cosmological problems etc. On the other hand, if some deviations in flavor physics are true, there should also be light sfermions. For example, the deviation of the muon anomalous magnetic dipole moment (muon $g-2$) from the SM prediction, $\delta(\frac{g-2}{2}) = (26.1 \pm 8.0) \times 10^{-10}$ [5, 6], is above the 3σ level. To relax the tension, a smuon as light as $\mathcal{O}(300)$ GeV is needed in the MSSM [7].

A split spectrum commonly has the naturalness problem proposed by 't Hooft [8]. If the particle mass is naturally small, by the definition of naturalness, there should be an approximate symmetry dominantly broken by the mass. The split spectrum mostly considered is the mass split between gauginos [9] (gauginos with higgsino [10]) and all the scalars except for the Higgs boson. The split is natural since chiral symmetry forbids the mass term of a gaugino/higgsino. In SUSY SMs, the sfermion mass comes from SUSY breaking, hence the lighter sfermions should be protected from the SUSY breaking by a symmetry while the heavier ones are not¹.

In this paper, we suggest a natural split mechanism using $N=2$ SUSY [12]. The non-renormalization theorem of $N=2$ SUSY forbids the sfermion mass terms which are embedded in $N=2$ multiplets even with SUSY slightly broken (Sec.2). If we add a sector with $N=1$ SUSY weakly coupled to the $N=2$ sector, the SUSY breaking dominantly affect the sfermion masses in the $N=1$ sector, while sub-dominantly to those in the $N=2$ sector since they are protected by $N=2$ SUSY. Therefore, a partly $N=2$ SUSY model can generate a naturally splitting sfermion spectrum.

¹Note that the split between sfermions seems to be unconstrained by the anthropic principle. This is the difference from the case of the electroweak scale and the cosmological constant [11].

As an example, we construct a natural split gauge-mediated SUSY breaking (GMSB) model [13] to explain the muon $g - 2$ anomaly within the 1σ level. We introduce the hyperpartners of left-handed sleptons, smuons, and down-type Higgs, with the extension of the $U(1)_Y$ gauge interaction to $N=2$ SUSY by introducing a singlet chiral multiplet. To cancel the gauge anomaly, we introduce spectator fields which are the mirror particles of the hyperpartners. The messenger sectors are also partly extended to $N=2$ SUSY so that the natural split mechanism works. The condition of gauge coupling unification (GCU) at the SUSY GUT scale, $\sim 10^{16}\text{GeV}$, naturalness, and the symmetries require the additional charged fields to be at the same scale, the messenger scale. Below the messenger scale, the MSSM particles and the singlet survive.

In our model, there are only three free parameters: the messenger scale, the dominant and sub-dominant F -terms. The dominant one only gives masses to the sfermions in the $N=1$ sector at the leading order due to the natural split mechanism. The sub-dominant one acts as the one in the usual GMSB model. We show an IR spectrum containing various testably light MSSM sparticles such as smuons, left-handed sleptons, and gauginos. In addition, there is a new light singlet fermion, bini, which is the $SU(2)_R$ partner of the bino. In our model, the bini mass is $\mathcal{O}(10)$ MeV, which is 2-loop suppressed to the bino mass. The light bini rarely interacts with the MSSM particles, due to the higher dimensional interactions suppressed by the messenger scale.

2 Natural Split Mechanism

A sfermion mass term comes from the bilinear Kähler term,

$$\mathcal{K} = \frac{|Z|^2}{M^2} Q^\dagger Q \supset |\theta|^4 \left| \frac{F_Z}{M} \right|^2 \tilde{Q}^\dagger \tilde{Q}. \quad (1)$$

We will show $N=2$ SUSY forbids such a term.

An $N=2$ SUSY extension of the SM is hardly considered for phenomenology. There are at least two difficulties. The renormalizable $N=2$ SUSY theory only has fermions that are in real representations, while the SM fermions are chiral. The Landau-pole of the gauge coupling is generated at a low energy scale. However, a partly $N=2$ SUSY extension at a high energy scale is not so problematic.

2.1 Review of $N=2$ SUSY

Here we briefly review $N=2$ SUSY. For simplicity, we consider $N=2$ SUSY quantum electrodynamics.

The Lagrangian is given in the $N=1$ superfield formalism as

$$\begin{aligned}
\mathcal{L}_{N=2} &= \mathcal{L}_{matter} + \mathcal{L}_{gauge}, \\
\mathcal{L}_{matter} &= \int d^4\theta \sum_i^{N_f} (\bar{\Phi}_i e^{-2Y_i V} \bar{\Phi}_i^\dagger + \Phi_i^\dagger e^{2Y_i V} \Phi_i) \\
&\quad + \int d^2\theta \sum_i^{N_f} (\sqrt{2} \bar{Y}_i \bar{\Phi}_i \Phi_Y \Phi_i + \sum_j m_{i,j} \bar{\Phi}_i \Phi_j) + \text{h.c.}, \\
\mathcal{L}_{gauge} &= (\frac{1}{g^2} \int d^4\theta \Phi_Y^\dagger \Phi_Y + \frac{1}{4g^2} \int d^2\theta W_\alpha W^\alpha + \text{h.c.}),
\end{aligned} \tag{2}$$

with $Y_i = |\bar{Y}_i|$, $[m^\dagger, m] = 0$. The Lagrangian is the most general non-trivial renormalizable Lagrangian with $N=2$ SUSY without the Fayet-Iliopoulos (FI) term [14].

There are two kinds of $N=2$ multiplets in the Lagrangian: vector multiplet and hypermultiplet. The Lagrangian has a specific symmetry, $SU(2)_R$ symmetry, that rotates the fields in the $N=2$ multiplet as in Tab.1. The $SU(2)_R$ symmetry is not

$N=2$ Multiplet	in Superfields		in Fields		renormalized at
Vector multiplet	(V, Φ_Y)		$((A^\mu, \lambda, D), (\Phi_Y, \psi_Y, F_Y))$		1-loop level
Hypermultiplet	$(\Phi_i, \bar{\Phi}_i^\dagger)$		$((\phi_i, \psi_i, F_i), (\bar{\phi}_i, \bar{\psi}_i, \bar{F}_i)^\dagger)$		tree level
$SU(2)_R$ Multiplet	$(\phi_i, \bar{\phi}_i^\dagger)$	(λ, ψ_Y)	(\bar{F}_i^\dagger, F_i)	$(\Re[F_Y], \Im[F_Y], \frac{1}{\sqrt{2}}D)$	the others
Representation	2	2	2	3	1

Tab. 1: $N=2$ multiplets and $SU(2)_R$ representations

only non-commutative with θ , but also not closed in an $N=1$ supermultiplet.

Since the Lagrangian has the $SU(2)_R$ symmetry only if $Y_i = |\bar{Y}_i|$, the $SU(2)_R$ symmetric models do not allow \bar{Y}_i to vary even with the wave functions renormalized. Because of the non-renormalization theorem for the superpotential, the $SU(2)_R$ symmetry leads to the non-renormalization theorem to the Kähler potential as in the Tab.1.

2.2 $N=2$ SUSY breaking

We consider the case that $N=2$ SUSY is slightly broken to $N=0$ by an F -term, F_Z , of a SUSY breaking field, Z . Since $N=2$ SUSY is recovered with the vanishing F -term, the effective Lagrangian should be

$$\begin{aligned}\mathcal{L}_{\text{eff}} &= \mathcal{L}_{N=2} + \mathcal{L}_{N2B}, \\ \mathcal{L}_{N2B} &= \int d^4\theta \tilde{W}(\Phi_i, \bar{\Phi}_i, \Phi_Y, Z, Z^\dagger) + \text{h.c.}\end{aligned}\tag{3}$$

Here \tilde{W} is a holomorphic function of $\Phi_i, \bar{\Phi}_i$, and Φ_Y . Therefore, even with SUSY breaking, sfermion masses like Eq.(1) are forbidden by the non-renormalization theorem. On the other hand, integrating $\bar{\theta}^2$ out, an effective superpotential, $F_Z^\dagger \partial_{Z^\dagger} \tilde{W}$, is generated.

If Φ_Y is a gauge singlet as in Sec.2.1, the dangerous tadpole term, $\delta\tilde{W} \sim Z^\dagger \Phi_Y$, may be generated. $Z^\dagger \Phi_Y$ can be a source of large $\Phi_Y \sim Z$ or $F_Y \sim F_Z$ unless fine-tunings. To have $\langle \Phi_Y \rangle \sim 0$ naturally, there are two possibilities. One is to introduce a large SUSY preserving mass term, $W = M\Phi_Y^2$, by hand to suppress $\langle \Phi_Y \rangle$. The other is to impose a symmetry to forbid the tadpole term or to have a naturally small one. In this paper, to forbid the tadpole term we assume a symmetry, under which Z is charged, while Φ_Y is not.

A concrete Z_2 symmetric superpotential with slightly broken $N=2$ SUSY is as follows.

$$W = \sqrt{2}Y\Phi_Y(\Phi_1\bar{\Phi}_1 + \Phi_{-1}\bar{\Phi}_{-1}) + Z(\Phi_1\bar{\Phi}_1 - \Phi_{-1}\bar{\Phi}_{-1}).\tag{4}$$

The model possess a Z_2 symmetry which exchanges the indices and reverses the sign of Z . Since Z can be identified as the chiral component of an $N=2$ vector multiplet¹ F_Z breaks the $SU(2)_R$ to $SO(2)$, which also forbids the sfermion mass terms, Eq.(1).

2.3 Natural split mechanism

We explain the essence of the natural split mechanism. Suppose we have three sectors²: SB sector, L sector and H sector. SUSY is broken in the SB sector, while

¹Actually, non-linear $N=2$ abelian gauge theory with the electric and magnetic FI terms aligned in the $SU(2)_R$ basis, can spontaneously break $N=2$ SUSY to $N=0$ with the chiral component of the $N=2$ vector multiplet acquiring SUSY breaking VEV, Z [15].

²A sector means that it is separated if some weak couplings are taken to zero.

the other sectors do not directly couple with the SUSY breaking. At the SUSY limit the SB and L sectors have $N=2$ SUSY, while the H sector has $N=1$ SUSY.

To have the SUSY breaking in the L sector, the SUSY breaking effect should be mediated firstly to the H sector to pick up the explicit $N=2$ SUSY breaking, and then be mediated to the L sector. The sfermion masses in the L sector are suppressed to those in the H sector. Therefore, integrating out the SB sector, we obtain a split spectrum for sfermions. If the explicit breaking of $N=2$ SUSY is small enough, the $SU(2)_R$ partners of the light particles are also light, which may imply new light particles. The natural split mechanism is summarized in Fig.1.

The set-up is natural, if the L and SB sectors have small deviations from the $SO(2)$ relations, in the sense of Eq.(4). Then, there are two explicit breaking sources of the $SO(2)$: the deviations and the existence of the H sector. It is natural (or stable under the quantum correction), when the two sources are the same order. For example, if the H sector particles contribute to the reactions in the L and SB sectors via loop effects as the case we will consider, it is still natural to have the deviations of the $SO(2)$ in the L and SB sectors as small as the loop effects. In the case, the split spectrum for sfermions is not spoiled since the sfermion masses from the $SO(2)$ deviations in the L and SB sectors is also suppressed.

The assumption of partly $N=2$ SUSY seems to be reasonable. This is because a series of SUSY theories have the possibility of partly $N=2$ SUSY as an effective theory. It may be obtained explicitly or spontaneously.

The compactified extra-dimensional theory with “our world” localized on an orbifold can be the former case. For example, $N=1$ SUSY on $R_{1,3} \times S_2/Z_2$ spacetime is effectively $N=2$ SUSY which is broken by Z_2 projection [16]. If “our world” is localized on the singularity, only Z_2 even fields survive at the low energy limit. Ordinary, to get $N=1$ SUSY, the $N=2$ partner of the MSSM particles are assumed Z_2 odd [10, 17]. However, our world may contain a part of a completely Z_2 even $N=2$ multiplet whose partner also survives at the limit. Therefore, partly approximate $N=2$ SUSY is obtained.

We may also break $N=2$ SUSY to $N=1$ spontaneously since $N=1$ gauge theory can be described by non-linear realized $N=2$ SUSY with chiral matters of any representations [18]. In [15], an $N=2$ non-linear abelian gauge model is shown to have such breaking with electric and magnetic $N=2$ FI terms in a generic case. Therefore, a separated sector, which does not directly couple to the $N=2$ gauge fields, has $N=2$ breaking only at the higher order.

Although the consistency of the above possibilities need to be investigated concretely in our future work, we assume a situation that $N=2$ SUSY is broken partly by some underlying physics in this paper.

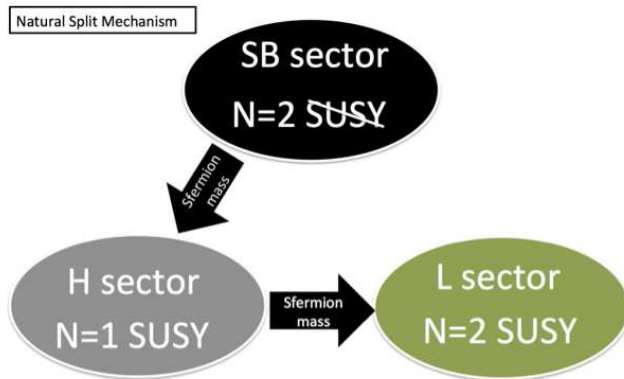


Fig. 1: The sketch of the natural split mechanism

3 Natural Split GMSB Model

In this section, we show a concrete model with the natural split mechanism.

We assume the effective MSSM, GCU at the SUSY GUT scale $\sim 10^{16}\text{GeV}$, and the explanation of the muon $g - 2$ anomaly within the 1σ level [5, 6, 7]. A GMSB model is a natural candidate to satisfy the assumptions, however as noted in [19], a split spectrum is difficult to obtain due to the non-zero hypercharge of the messengers except for those belonging to the adjoint representation of $SU(5)$. The natural split mechanism can generate a split spectrum even with only hypercharged messengers.

One of the advantages of the GMSB mechanism in our case is that the ordinary messenger fields are able to be the hyperpartners of the muon superfields. The other advantage is as follows. To keep light smuons natural in the effective MSSM, the following relations are required from the 1-loop renormalization group equations

(RGEs) [20].

$$m_{\text{smuon}}^2 \gtrsim (Y g'^2 S, Y^2 g'^2 M_1^2, g^2 M_2^2), \quad (5)$$

$$\text{where } S = \sum_i^{\text{DOF}} m_i^2 Y_i = \text{Tr}[m_{\tilde{d}}^2 + m_{\tilde{e}}^2 - m_{\tilde{L}}^2 + m_{\tilde{Q}}^2 - 2m_{\tilde{u}}^2] - m_{H_d}^2 + m_{H_u}^2. \quad (6)$$

Since the vanishing of the right hand side of Eq.(5) implies that smuons are separated from the other sectors, the zero limit of the smuon masses is the recovery of SUSY in the separated smuon sector. Therefore, we should have a light wino, a light bino and a small S -term. To obtain a small S -term, there should be a large cancellation of the heavy sfermion masses including the stop masses $\gtrsim \mathcal{O}(10)$ TeV. In the GMSB mechanism, the S -term is guaranteed to vanish automatically [21]. The gauginos are naturally light if the messenger sector has an approximate discrete R symmetry.

3.1 The set-up of the natural split GMSB model

The Lagrangian density just above the messenger scale, M , is as follows.

$$\begin{aligned} \mathcal{L} = & \int d^4\theta (K_{\text{MSSM}} + K_{\text{N1Mess}} + K_{\text{N2Mess}} + K_{\text{Partner}} + K_{\text{Spectator}}) \\ & + \int d^2\theta (W_{\text{MSSM}} + W_{\text{N1Mess}} + W_{\text{N2Mess}} + W_{\text{N2}} + W_{\text{mass}}) \\ & + \text{h.c.} + \mathcal{L}_{\text{gauge}}, \end{aligned} \quad (7)$$

$$K_{\text{MSSM}} = \sum_{i=1}^3 (|Q_i|^2 + |u_i|^2 + |d_i|^2 + |L_i|^2 + |e_i|^2) + |H_u|^2 + |H_d|^2, \quad (8)$$

$$K_{\text{N1Mess}} = (|\Phi_u|^2 + |\bar{\Phi}_u|^2 + |\Phi_Q|^2 + |\bar{\Phi}_Q|^2), \quad (9)$$

$$K_{\text{N2Mess}} = \sum_a^{+, -} \sum_i^{-1, 1} (|\Phi_{di}^a|^2 + |\bar{\Phi}_{di}^a|^2), \quad (10)$$

$$K_{\text{Partner}} = \frac{1}{g'^2} |\Phi_Y|^2 + |\bar{e}_2|^2 + |\bar{H}_d|^2 + \sum_{i=1}^3 |\bar{L}_i|^2, \quad (11)$$

$$K_{\text{Spectator}} = |S_{e2}|^2 + |S_{Hd}|^2 + \sum_{i=1}^3 |S_{Li}|^2, \quad (12)$$

$$W_{\text{MSSM}} \sim y_t H_u Q_3 u_3 + y_b H_d Q_3 d_3 + y_\tau H_d L_3 e_3, \quad (13)$$

$$W_{\text{N1Mess}} = \lambda_u Z \Phi_u \bar{\Phi}_u + \lambda_Q Z \Phi_Q \bar{\Phi}_Q, \quad (14)$$

$$W_{\text{N2Mess}} = \sum_a^{+, -} \left(\sum_i^{-1, 1} \left(\frac{1}{\sqrt{2}} \frac{1}{3} \Phi_Y \Phi_{di}^a \bar{\Phi}_{di}^a \right) + Z_a (\Phi_{d1}^a \bar{\Phi}_{d1}^a - \Phi_{d(-1)}^a \bar{\Phi}_{d(-1)}^a) \right), \quad (15)$$

$$W_{N2} = -\frac{1}{\sqrt{2}} \int d^2\theta \Phi_Y \left(\frac{1}{2} \sum_{i=1}^3 L_i \bar{L}_i + \frac{1}{2} H_d \bar{H}_d - e_2 \bar{e}_2 \right) + \text{h.c.}, \quad (16)$$

$$W_{\text{mass}} = M \left(\sum_i^3 \bar{L}_i S_{Li} + \bar{H}_d S_{Hd} + \bar{e}_2 S_{e2} \right), \quad (17)$$

where

$$\begin{aligned} Z &= M + \theta^2 F = M(1 + \theta^2 \Lambda), \\ Z_+ &= M + \theta^2 F_d = M(1 + \theta^2 \Lambda_d), \\ Z_- &= M - \theta^2 F_d = M(1 - \theta^2 \Lambda_d), \end{aligned} \quad (18)$$

with $\Lambda_d \gg \Lambda$ and $\lambda_{u,Q} \sim \mathcal{O}(1)$. Here, we omit to write down the terms with $N=1$ vector multiplets. In the model, the $U(1)_Y$ gauge interaction are partly extended to $N=2$ SUSY with introducing the $N=2$ partners: \bar{L}_i , \bar{e}_2 , \bar{H}_d and Φ_Y . The spectator fields, S_{Li} , S_{e2} and S_{Hd} , are needed to cancel the gauge anomalies and to obtain the mass terms of the introduced hyperpartners to satisfy the assumption of effectively MSSM. There are two messenger sectors: the ordinary one and the one with the $U(1)_Y$ gauge couplings extended to $N=2$. The latter one couples with SUSY breaking fields, Z_+ and Z_- , with the F -term much larger than that of the former one, Z . The doubling of Z_+ and Z_- are needed in order to cancel the large gaugino masses by an approximate Z_{4R} symmetry¹. To forbid the tad-pole term of the $N=2$ $U(1)_Y$ vector partner, Φ_Y , namely sb, a precise Z_2 symmetry is also needed. The components and their properties are shown in Tab.2.

We explain the reasons for the choice of the hyperpartners and the SUSY mass terms. The choice of $N=2$ sector is not artificial, but is the requirement of naturalness, which urge us to have a small S -term. The would-be large smuon masses mediated by the $U(1)_Y$ should be almost cancelled by $N=2$ non-renormalization theorem as in Sec.2.2. The GMSB effect via the V_Y to a scalar mass is $\delta m_i^2 \propto Y_i^2$, hence the S -term is generated from the zero value of the GMSB mechanism: $\delta S \propto -\sum_i Y_i^3 = -2Y_{L2}^3 - Y_{e2}^3 = -\frac{3}{4}$. To cancel the S -term, one of the solution is to have three additional cancellations of the $(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$ -type scalar masses. Therefore, we

¹Such an $N=2 \rightarrow N=0$ SUSY breaking model can be realized if we have two Z_2 related $N=2$ abelian vector multiplet systems in [15] with the same prepotentials but opposite FI terms. Since the $N=2$ D -term is proportional to the FI term and the scalar potential does not care about the sign of the FI term, the two F -terms of the chiral components are opposite while the scalar VEVs, if no degeneracy, are the same. If Φ_{di}^+ and Φ_{di}^- couple with the two $N=2$ vector multiplets respectively, the Z_+ and Z_- can be recognized as the VEVs of the chiral components of them.

N=2 sector	MSSM particle and N=2 partner				Messenger	Spectator		
Component	V_Y, Φ_Y	$L_i, \overline{L}_i^\dagger$	$H_d, \overline{H}_d^\dagger$	$e_2, \overline{e}_2^\dagger$	$\Phi_{di}^a, \overline{\Phi}_{di}^{a\dagger}$	S_{Li}	S_{Hd}	S_{e2}
SM Gauge	$[1, 1, 0]$	$[1, 2, -\frac{1}{2}]$		$[1, 1, 1]$	$[\overline{3}, 1, \frac{1}{3}]$	$[1, 2, -\frac{1}{2}]$		$[1, 1, 1]$
DOF	/	3×2	1×2	1×2	2×2×2	3	1	1

N=1 sector	MSSM chiral field					Messenger			
Component	Q_i	u_i	d_i	e_1, e_3	H_u	Φ_Q	$\overline{\Phi}_Q^\dagger$	Φ_u	$\overline{\Phi}_u^\dagger$
SM Gauge	$[\mathbf{3}, \mathbf{2}, \frac{1}{6}]$	$[\overline{\mathbf{3}}, \mathbf{1}, -\frac{2}{3}]$	$[\overline{\mathbf{3}}, \mathbf{1}, \frac{1}{3}]$	$[\mathbf{1}, \mathbf{1}, 1]$	$[\mathbf{1}, \mathbf{2}, \frac{1}{2}]$	$[\mathbf{3}, \mathbf{2}, \frac{1}{6}]$		$[\overline{\mathbf{3}}, \mathbf{1}, -\frac{2}{3}]$	
DOF	3	3	3	2	1	1	1	1	1

Action under Describe Symmetry							
Field	Φ_Y	Z_\pm	Z	$(\Phi_{di}^a, \overline{\Phi}_{di}^a)$	$\overline{\Phi}_Q$	$\overline{\Phi}_u$	Others
Z_2	Φ_Y	$-Z_\pm$	$-Z$	$(\Phi_{d(-i)}^a, \overline{\Phi}_{d(-i)}^a)$	$-\overline{\Phi}_Q$	$-\overline{\Phi}_u$	Itself
Z_{4R}	$\Phi_Y(i\theta)$	$Z_\mp(i\theta)=Z_\pm(\theta)$	$Z(i\theta)$	$i(\Phi_{di}^{(-a)}, \overline{\Phi}_{di}^{(-a)})$	$i\overline{\Phi}_Q$	$i\overline{\Phi}_u$	$=\sqrt{Z_R}$

Tab. 2: Particles and symmetries in the $N=1$ language. Z_R is R -parity.

can have the hyperpartners as in Tab.2 at least. Suppose that the hyperpartners have Dirac mass terms with the spectators. Then the hyperpartners should decouple almost at the same scale, otherwise the cancellation of the S -term is spoiled by the RGEs. Therefore, three SUSY masses, M , Z and Z_\pm , should be the same order², when we assume the GCU at the SUSY GUT scale, since the hyperpartners, the messengers and the spectators can be embedded into complete $SU(5)$ multiplets, $(\mathbf{5} + \overline{\mathbf{5}}) \times 4$ and $\mathbf{10} + \overline{\mathbf{10}}$. Note that as usual GMSB, the sfermion mass squared depends on Λ^2, Λ_d^2 linearly but the SUSY masses logarithmically accompanied by a loop factor [13]. Therefore, the change of the relative factor ~ 3 between the SUSY masses, is sensitive only up to the next loop order to the sfermion masses. This allows us to set the SUSY masses, M , Z and Z_\pm , to be the same value, M , without changing the result at the leading order³. There is a bound for M to be larger than $\mathcal{O}(10^6)\text{GeV}$ so that the GCU remains perturbative.

²Since $M \lesssim Z_\pm$, we can show that in general, the GCU is obtained at a scale $\lesssim 10^{16}\text{GeV}$. We have taken a most favored value by the proton decay problem.

³The bini mass actually has a power-law dependence on the relative factor, but in this paper we do not care about its value but the order.

3.2 Natural split mechanism and low energy spectrum

If we forget about the spectators and the $N=1$ messengers, the model corresponds to a complicated case of Fig.1. We can identify that the $N=2$ messenger sector is the SB sector, the hypermultiplets in the $N=2$ sector compose the L sector, the $N=1$ and $N=2$ gauge multiplets are mediators, and the other fields compose the H Sector.

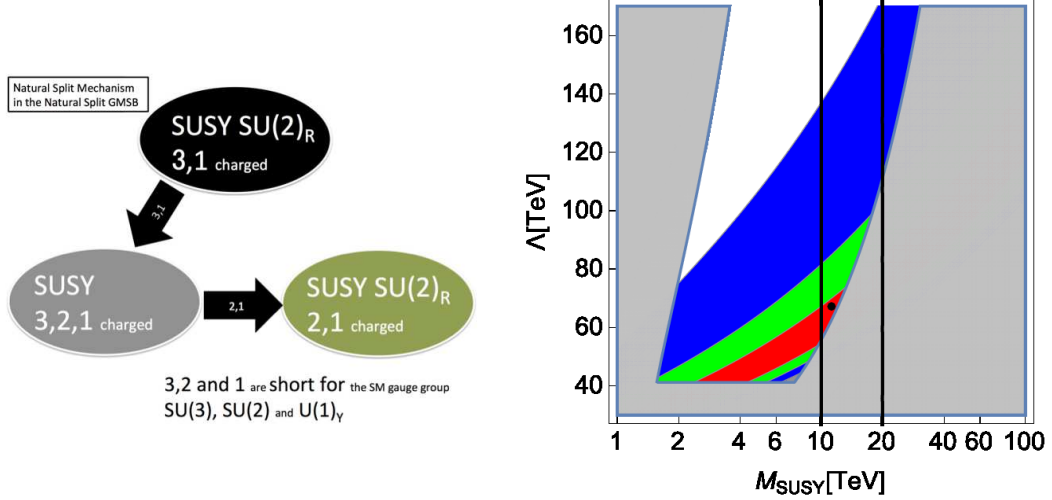


Fig. 2: **Left** The sketch of the mediation of the dominant SUSY breaking, F_d . The black (grey, green) ellipse is the SB (H, L) sector. **Right** The parameter region of the natural split GMSB model with $\tan\beta=7$, $M = 10^{10}$ GeV. Between the two vertical black-lines, the Higgs boson mass is between 124.5 GeV and 127.2 GeV calculated by FeynHiggs 2.11.2 [23] with about 1 GeV uncertainty. M_{SUSY} is the geometric average of the stop mass. The red (green, blue) region explains the muon $g - 2$ anomaly within the 1σ (2σ , 3σ) level. The black dot has the information in Tab.4.

The natural hierarchy is obtained as in Tab.3 by integrating out the degrees of freedom above the messenger scale. The gauge charges and the natural split mechanism forbid SUSY breaking to mediate into the L sector at the leading order. The sfermion masses in the L sector are naturally suppressed to those in the H sector which is mediated directly from the SB sector. The gauginos are also naturally light since the Z_{4R} parity forbids the gaugino masses, as well as the naturalness of small F compared with F_d .

With solving the RGEs of the MSSM, we show the allowed region in Fig.2 using the method developed in [22]. The black dot in the allowed region (Fig.2) has the IR spectrum in Tab.5. In our analysis, we treat B_0 as a free parameter as well as μ_0 .

Sector	H Sector			Mediator Sector				
Particle	Squarks	\tilde{e}_1, \tilde{e}_3	H_u	Sb	Bini	Bino	Wino	Gluino
Mass $\frac{g'^2}{16\pi^2}$	$\sqrt{\frac{20}{3}} \frac{g_3^2}{g'^2} \Lambda_d$	$\sqrt{\frac{10}{3}} \Lambda_d$	$\sqrt{\frac{5}{6}} \Lambda_d$	$\frac{\sqrt{32}}{3} \frac{g_3}{g'} \Lambda_d$	$L^2 \Lambda$	$\frac{3}{2} \Lambda$	$\frac{3}{2} \frac{g^2}{g'^2} \Lambda$	$\frac{3}{2} \frac{g_3^2}{g'^2} \Lambda$
Sector	L Sector							
Particle	$\tilde{L}_1, \tilde{L}_2, \tilde{L}_3, H_d$					\tilde{e}_2		
Mass ²	$3(\frac{g^2}{16\pi^2})^2 \Lambda^2 + \frac{3}{4}(\frac{g'^2}{16\pi^2})^2 \Lambda^2 + Lm_{H_u}^2$					$3(\frac{g'^2}{16\pi^2})^2 \Lambda^2 + Lm_{\tilde{e}_{1,3}}^2$		

Tab. 3: The formula of mass spectrum just below the messenger scale M with assuming $\Lambda_d \gg \Lambda$ and neglecting the Λ compared with Λ_d [13]. L denotes the loop factor $\sim \mathcal{O}(10^{-2})$ which can be determined by the higher loop calculations.

This is because there are many possibilities to generate it whether relating to the μ problem or not. For example, we can consider that a sequestered sector has SUSY breaking of $\frac{F_{\text{seq}}}{M_{\text{pl}}} \sim \mathcal{O}(1)\text{TeV}$ and the supergravity potential automatically contains $B_0 \sim \frac{F_{\text{seq}}}{M_{\text{pl}}}$. It is confirmed by FeynHiggs 2.11.2 [23] that the spectrum in Tab.5 leads to the Higgs boson mass, $m_h = 125.7 \text{ GeV}$, and the SUSY contribution to the muon $g-2$, $\delta(\frac{g-2}{2}) = 2.6 \times 10^{-9}$. The GCU is obtained at the SUSY GUT scale perturbatively, with $\alpha_{\text{GUT}}^{-1} \sim 8-10$. Therefore, the model can explain the muon $g-2$ anomaly naturally within the 1σ level and the Higgs boson mass with $\mathcal{O}(10)$ TeV stops. If the stop masses are heavier than 20 TeV, the calculation here is not reliable since the mediation of Λ_d to the L sector via the H sector (Fig.2) is not negligible compared with the mediation of Λ . Naively, this does not occur when

$$m_{\tilde{e}_2}^2 > \frac{1}{16\pi^2} m_{\tilde{e}_3}^2. \quad (19)$$

The condition also guarantees the neglecting of the other sub-dominant factors, such as the 2-loop RGE effects, the natural deviation of couplings from the $N=2$ SUSY relations (Sec.2.3), and up to factor ~ 3 differences between the SUSY masses M , Z and Z_d (Sec.3.1). This is the constraint for the right boundary of the allowed region in Fig.2.

Parameter	$\tan \beta$	M	Λ_d	Λ	μ_0	B_0
Value	7	10^{10} GeV	930 TeV	68 TeV	12 TeV	1.5 TeV

Tab. 4: The fundamental parameters corresponding to the black dot in Fig.2

H sector	Mass	L sector	Mass
\tilde{Q}_3	12.5 TeV	\tilde{L}_3	538 GeV
\tilde{u}_3	10.9 TeV	\tilde{L}_1, \tilde{L}_2	544 GeV
$\tilde{d}_1, \tilde{d}_2, \tilde{d}_3$	14.1 TeV	\tilde{e}_2	216 GeV
\tilde{e}_1, \tilde{e}_3	2.6 TeV	Mediator	Mass
\tilde{Q}_1, \tilde{Q}_2	14.1 TeV	\tilde{G}	1940 GeV
\tilde{u}_1, \tilde{u}_2	14.2 TeV	\tilde{W}	543 GeV
A-Higgs	11.4 TeV	\tilde{B}	165 GeV
Higgsino	11.3 TeV	bini	$\mathcal{O}(10)$ MeV
Higgs (by FH [23])	125.7 GeV	sb	2600 GeV

Tab. 5: The low energy spectrum corresponding to the black dot in Fig.2

There are six fundamental parameters, $(M, \tan \beta, \Lambda_d, \Lambda, \mu_0, B_0)$, and three equalities: the Higgs boson mass and the electroweak symmetry breakdown conditions. Hence, three free parameters remain while the number of the sparticles lighter than 2 TeV are seven. Therefore, the model is predictive. We note that the predictability holds as long as Eq.(19) is satisfied roughly, otherwise not only higher loop effects but also many subdominant parameters, which we have neglected, become effective. In our fundamental parameter set in Tab.4, we can show only the mass of \tilde{e}_2 may alter at a factor of $\lesssim \mathcal{O}(1)$ from our prediction due to the closeness to the bound, Eq.(19), while the other masses change at most by $\mathcal{O}(1)\%$ from a naive estimation of higher loop effects. Another feature of the spectrum is the heavy 1st/3rd generation right-handed sleptons. The model is favored by the CP bounds of the 1st generation fermion and the vacuum stability of the stau.

The model predicts new singlet boson and fermion: sb and bini. The effective couplings between the sb and the MSSM superfields are proportional to $\frac{1}{M^2}$ at the SUSY limit due to the Z_2 parity. The suppressed couplings imply the difficulty to detect them in experiments and the ease to be free from experimental problems. Cosmologically, the sb decay may spoil the successful BBN prediction. To avoid this, there are two possibilities: the cases of a low enough reheating temperature (or a high enough messenger scale) and of a low enough messenger scale. The former one prevents thermal generation of the sb while the latter one advances the decay before the BBN.

We have not considered the constraints from cosmology in choosing the spectrum.

Naively, the spectrum is not viable cosmologically, at least the thermal abundance of the bino-like neutralino is over-produced. However, R -parity violations (in which case the bini can be the dark matter), coannihilations with sfermions taking the higher corrections into account, dilutions via decay of a scalar field (which can be the sb) etc., can cure the problem. The cosmological implications in detail will be studied in our future work.

4 Conclusions

In the paper, we consider the situation that the SUSY scale is far beyond the EW scale, while some of the sfermions are light enough to deviate the measured values of some flavor physics from the SM predictions.

We suggest a natural split mechanism using $N=2$ SUSY. We show that $N=2$ SUSY protects the Kähler kinetic term of a hypermultiplet from quantum corrections even when SUSY is slightly broken. Therefore, partly $N=2$ SUSY models realize the natural split spectrum of sfermions, if some of the sfermions are contained in the hypermultiplets. The situation may be feasible in some underlying theories.

We assume at a scale partly $N=2$ SUSY is realized, and build a natural split GMSB model that explains both the muon $g-2$ anomaly at the 1σ level and the measured Higgs boson mass. The model contains three free parameters and seven testably light sparticles with the masses below 2 TeV. In addition there is a new light fermion, bini. The bini has the mass of MeVs, which is 2-loop suppressed compared with the bino mass. The bini is experimentally safe due to the smallness of the effective couplings.

Therefore, we have shown that a partly $N=2$ SUSY extension is a natural split mechanism for sfermions, and the light sfermion, if detected, is not only a sign of $N=1$ SUSY but also may be a sign of $N=2$ SUSY.

5 Discussions

Let us discuss about the assumptions we have made to build the natural split GMSB model. We have assumed the following conditions for the sake of clarity as a good example of the natural split mechanism.

We have chosen the additional fields to the MSSM that can be put into complete $SU(5)$ multiplets. However, there are other choices of fields whether preserving GCU

at the SUSY GUT scale or not.

We have assumed that the low energy spectrum is the MSSM-like and is calculated by using the RGEs of the MSSM, which constrains the UV spectra into ones that are stable under the RGEs from the viewpoint of naturalness. However more various stable UV spectra can exist, especially if some of the $N=2$ partners do not decouple, since a part of the $SU(2)_R$ would cancel some of the RGE corrections. In the case, the experiments constrain the model more severely, but plenty of unique signatures should be possible to predict.

As explained in the Sec.2.3, we should have loop suppressed deviations from the exact $SO(2)$ relations in the L and H sectors from the viewpoint of naturalness. The sfermion mass corrections due to the deviations are the same order as the ones via the H sector in the loop expansion, hence if the expansion is valid, the effects are neglected due to Eq.(19). However, we have to face with the breakdown of the validity of loop expansion due to the hierarchy of couplings and flavors. In our case, we can not put all the missing partners just above our messenger scale due to the assumption of the GCU. If we put the missing partners at the SUSY GUT scale and assume the partly $N=2$ SUSY condition there, the $N=2$ relations would be broken due to the invalidity of the loop expansion after resummation of the logarithms by using the RGEs of Eq.(7). This problem can be solved by setting the messenger scale near the SUSY GUT scale in which case the stop masses are reduced to $\sim 7\text{TeV}$ to solve the muon $g - 2$ anomaly within 1σ level which is less favored in the light of the CP, flavor and proton decay problems. Also perturbativity does not constrain the additional particle contents any more. We can also assume some underlying theory above the messenger scale to modify the RGEs, however in the case we have to answer why the theory does not change the predictions of the model. Therefore, this is a naturalness problem above the messenger scale. Conversely, this is also a hint to constrain the additional particles, since our model is not the only viable one.

In the paper, from the viewpoint of naturalness at and below the messenger scale, we have built one of the viable models. However, the underlying theory and the naturalness above the messenger scale should further constrain the additional particle contents. And, improvements will be shown in our future works as well as their phenomenology and cosmology in detail.

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